§4.1 # Chapter Four

Concerning Trisection.

For if the subtended chord of any arc is given, and the subtended chord of the third [part] is sought, it is permitted to use the same source for our inspiration[1] as previously, nevertheless the discovery is made with a little more labour.

Let the given subtending chord of 36 Degrees be 061803398875. Three of the Subtended Chords of 12:0': Degrees are equal to the sum of the given subtended chord of 36:0' and the cube of the subtended chord 12:0', *by the second conclusion of the preceding chapter*. This equation is expressed thus[2]:

$$3① \ = \ 061803398875 + 1③ \ \text{or} \ \ 3① \ - 1③ = 061803398875$$

1. Given therefore the subtended chord divided by three, and between the divisions always adding the cube of the quotient from that place [in the expansion of the number]; finally the quotient found will be the latus [i.e. root] sought, truly the third of the subtended chord sought. Therefore the positions of the points of the cubes [above the digits] are marked specially beginning from the place of unity, with two places put in between: as you see here.

$$\overset{.\quad.\quad.\quad.}{3)061803398875}$$

The first place towards the left is the position of unity ; surely the place occupied by the radius.

Then commencing with the division by the given third, which in the place of unity is not possible, or for the first time of being taken away : therefore a zero is placed in the quotient, and the amount to be put in the next place is sought: now 3 is found [i.e. the initial divisor]. This going twice into the nearby place, I place therefore 2 into the quotient, and of this quotient the cube is 8. This is added to the dividend at its own place, surely in the next place for the cube; and by taking away the total, the triple of the quotient is found as you see here.

$$\overset{.\quad.\quad.}{3)061803398875(02}$$

8	*Cube to be added*
0626	
6	*Triple of Quotient to be taken away.*
26033	*Remainder*
26033988(020	

[Table 4-1]

Then I progress asking how many times 3 goes into 2, the next place, and as it is not possible to take anything away, I place zero in the quotient, and I progress to the further remainder to be added, found in the next three subsequent places, as the more appropriate nearest gnomon[3] [or 'pointing number'] cube to that place is added; and now I seek how many times 3 goes into 26, which can be had eight times: but where by the gnomon the cube will have been increased in its place, the given divisor 3 can be taken away nine times. And so that we will be able to judge more properly which number should be placed in the [next] place in the quotient, with the product from the given divisor 3, and the product from the square of the quotient first found tripled (which not inconveniently will be called the *Divisor of the Cube*) is added in the same place to the Quotient : most conveniently it makes the divisor taken away smaller rather than being too large; as with the remaining numbers (which we may call the *Correct Divisor)* we are surer of the number in the Quotient that should be put in place. These divisors therefore

26033988	Remainder to be divided.	
3 . . .	Divisor[5].	
120000	Taken from Divisor.	
288 . .	Divisor corrected.	

40000	3	*Latus found 20.*
200	3	*Square of the latus found 400*
120000		
600		

By having found the Correct Divisor, I go on and ask how many times 288 [goes] into 2603.
And with finding the number in the Quotient should be placed 9.

26033988(0209	
1129329	With the gnomon of the Cube to be added†
27163317	
27 . . .	Product from root number 9 by 3 taken from the divisor
163317750	Remainder

120000	9
600	81
1080000	729
48600	
729	
1129329†	

[Table 4-2]

are brought together[4], because it can hardly be done except with both the one and the other put in its own place

This remainder gives 0 for the next place of the quotient, and if the divisors are brought together, the correct divisor is 2868 ¢ which gives the quotient 5 that should be written for the place after the zero.

163317750(0209056	
6553718	Gnomon of the Cube to be added*
169871468	
15 . . .	Three times the root taken away
19871468	
786657	Gnomon of the Cube to be added†
20658125	
18	Three times the root taken away
2658125	Remainder

4181	3000000	*Divisor given*
209056 *Root*	131043	*Triple the Square* §
40000	2868957	*Correct Divisor* ¢
3681		
43681†	*Square of the root 209*	
209025		
2508636		
43704411136	*Square of the roots 209056*	

†436810000	3
20900	3
§1310430000	5
62700	25
6552150000	125
313500	
125400	
125	
*6553717625	

437011902500	3
209050	3
13110557075	6
62715	36
7866342450	216
376290	
188145	
216	
†786656 | 822616	

[Table 4-3]

Another Example

2. Let the given subtended chord of 150:00': degrees be 1931851652578. The subtended chord of a third of the arc is sought, namely of 50:00': The equation is expressed thus:

3① = 1931851652578 + 1③

The first place of the root should be taken as large as possible. As here 8, which very much exceeds a third of the number 19 to be divided.

6400	3
80	3
†19200	4
240	16
76800	64
1440	
24	
64	
*80704	

Where indeed the cube, namely,512 is added to the dividend, in the second place, they make 2443 on addition. And from this number the triple of the quotient 24 is taken away. But if I should put 9 in the quotient, the cube of the quotient, on addition to the dividend yields 2660, from which the triple of the quotient 27 cannot be taken away[6].

```
19318516525578 (8
  512   Cube of the Quotient to be added.
2443
24  . . The triple of the root found  taken away.
    43851 Remainder
     3  . . Divisor
 † 19200  Taken from Divisor              8
    10800  Corrected divisor          64 Square.
```
                    [Table 4-4]

     The next place of the quotient is found thus. The product from the given divisor 3, in the nearest place of the quotient is taken away, and the product from the square of the root triplicated to give 19200, in the same place of the quotient (which has followed the particular part of the gnomon of the Cube): therefore the third given divisor is made smaller, by taking away of the triplicate of the square of the root found in the usual way, and collected together as you see above [Table 4-4], and that remainder is 10800, situated in the place of the divisor, which we can call the *corrected Divisor,* a method that scarcely ever fails in extracting the quotient; therefore in the division of 43851 by 10800, the quotient is 4, which gives the gnomon of the cube added 80704*.

```
  43851652 (8
  80704  .  .  .  Cube of the Gnomen added. *
124555
12  . . The triple of the root found taken away.
    4555652  Remainder
```
                    [Table 4-5]

The next place of the quotient is not found by the corrected dividend found above: but when the square of the root found is increased, because the triple must be taken away from the given divisor 3, it is necessary that the corrected divisor is diminished. As with the root found 84, the square 7056, of which the triple 21168: this is taken away from the three, thus

```
4555652
3 .  .  . Divisor taken away
2116800  Added to the Cube of the Divisor
08832 . . Corrected Divisor, which gives 5 in the division
 .  .  .
```
                    [Table 4-6]

```
   4555652                                          1690          3000000
Remainder (84523
   10647125        Gnomon of the Cube added*        8452          2142075
   15202777                                         64  .  .Sq.   0857925 Divisor
   15  .  .  .     Three times the root taken away   656
      202777578                                     7056†
      428516408    Gnomon of the Cube to be added†  8425
      631293986                                     714025  Sq.††
      6            Three times the quotient taken away   33804
         31293986  Remainder                        71436304 Sq. *
```

†705600	3	††71402500	3	*7143630400	3
840	3	8450	3	84520	3
2116800	5	214207500	2	21430891200	3
2520	25	25350	4	253560	9
10584000	125	428415000	8	64292673600	27
12600		101400		2282040	
5040		8		27	
125		428516408		64294955	667	
10647125					

[Table 4-7]

3. However this same line 19318516525578 subtends two arcs, the smaller one the semi-arc of 150:0': the remaining larger of 210:0': (since together they are equal to the whole periphery of the circle) a sub-tangent of third of this arc is sought, indeed of 70:0' degrees. The equation is the same as before:

$$3\,① \; = \; 1931851652578 + 1\,③$$

```
19318516525578 (1
1____    Cube to be added.
2931
3  .  . The triple of the root.
  068148347422   Remainder
```

[Table 4-8]

The remainder of three [times] the root exceeds the sum of the cube of the same root and the given subtending chord. Therefore for this remainder, three times the root is always added on to the dividend, and the gnomon of the cube taken away. As you see here[7].

In the next place, in the Quotient one has to be put 1, for if 2 is put in this place, the cubic gnomon is larger than it can possibly be to be taken from the remainder, and the triple of the quotient is added to the same.

```
   068(11                                  22              100 | 3
   3  .  .  .   3①  to be added.          11[47]           10 | 3
   368                                     1  . .          300 | 1
   §331.  . Cubic Gnomon to be taken away.    21            30 | 1
     37148  Remainder.                    †121 Square.       1 | 1
     36300  Cubic Divisor to be taken away.   896          §331
     3  .  .  . Given Divisor to be added  ††12996 Square.
       6300  Corrected Divisor.             16009
              [Table 4-9]                 *1315609 Square.
```

The corrected divisor 63 can be taken away 5 times from the remainder 371, but the total will be more than the gnomon that can be taken away; therefore let four be the figure placed in the quotient. The Gnomon to be taken away is 150544.

```
  37148(114                                      †12100 │ 3
  12 . . .   3①  to be added.                       110 │ 3
  157148                                           36300 │ 4
  *150544   Cubic Gnomon to be taken away.           330 │ 16
     6604347  Remainder.                           145200 │ 64
     3898800  Cubic Divisor to be taken away          1980 │
     3 . . .       Given Divisor to be added.          330 │
      898800  Corrected Divisor.                        64 │
                                                   *150544 │

  The corr.div.8988 seven times in the rem. 66043.                *131560900 │ 3
     6604347 Remainder. (1147              1299600 │ 3            11470 │ 3
     21 . . .   3①  to be added.              1140 │ 3        394682700 │ 1
     27604347                                3898800 │ 7          34410 │ 1
  † 27459523   Cubic Gnomon to be taken away.   3420 │ 49             1 │ 1
      144824422  Remainder. (11471          27291600 │ 343   §394717111 │
      3 . . .   3①  to be added.              30780 │
      444824422  Cubic Divisor taken away    136800 │           13158384100 │ 3
    §394717111 Cubic Gnomon to be taken..        343 │             114710 │ 3
       50107311 Remainder. (114715287270    †27459523 │        394751522300 │ 5
      15 . . .   3①  to be added.             2294                  344130 │ 25
       200107311                              11471             197375761500 │ 125
   ††197384364875                          *1315609.  Sq.          1720650 │
        2722946                              22941                   68826 . . │
                                            31583841   Sq.              125 │
                                                             ††197384364875
```

[Table 4-10]

The subsequent figures which you see here are added on being found by the same way. And the subtended chord of 70:0': degrees sought is 114715282702. And by this method we can extract the subtended chord of the third part if the given subtended chord of the triple.

4. But if the whole circle is added to the arc, of which the subtended chord shall be given, we can find the subtended chord of the third part of the total put together from the circle and the given arc. The subtended chord of 81:0': degrees, 1298896066660366 is given.

The sum of the arc given and the circle is 441 Degrees, 0': the third part of this sum is 147 Degrees, 0:' I wish to find the subtended chord of this arc. The cube of the subtended chord sought is equal to the sum of three times the root of the same cubic (or the subtended chord sought tripled) and of the given Subt. Ch. of 81:0': The equation is expressed thus:

3① + 129889 = 1③

Everything is set up as before[8].

1298896096660366 (1
3 . . . . *Three of the quotient added.*
4298
1 . . . *Cube of the quotient taken away.*
  3298 *Remainder*

[Table 4-11]

It is agreed by the proposition, that three times the quotient added to the given number equals the cube: nevertheless this will not happen until the whole quotient is found; which tripled and added to the given subtended chord is equal to the whole cube, with all the gnomons taken together.

```
                                                        ††100 │ 3
3298(19                                                   10 │ 3
27 . . .   Three of the quotient to added.               300 │ 9                  ¶36100 │ 3
5998                                                      30 │ 81                    190 │ 3
†5859 Gnomon of cubic taken away.                       2700 │ 729               *108300 │ 1
  139896 Remainder. (191                                2430 │                       570 │ 1
    3 . . .Three times quotient  added.                  729 │                         1 │ 1
  169896 Cubic divisor taken away                       †5859                      *108871
 *108871 Gnomon of cubic taken. 3 ...       added to given div.
    61025096 Remainder.              *108300   3 times Sq. of root subtr.
   3 . . . . To be added to given divisor.    78300      Corrected divisor   subtracted
  10944300 Cubic divisor taken away.
   7944300 Corrected divisor
```

```
 61025096 (1917                              38352                    ᶜ3648100 │ 3
  21 . . .    Three times quotient added.    191763                       1910 │ 3
 82025096                                    1 . . Sq.††               10944300 │ 7
ʳ76891213    Gnomon of cubic taken.          261                          5730 │ 49
   5133883660(19176                          361 Sq.¶                  76610100 │ 343
    18 . . .   Three times quotient added.   381                         51570 │
   6933883660                                36481 Sq.ᶜ                   2292 .. │
   †6616870776 Gnomon of cubic taken.        26789                         343 │
     3170128843669191763                     3674889 Sq.ᵛ             ʳ76891213 │
    9 . . .   Three times quotient to be added.  230076
    407012884366                             367718976 Sq.§           ᵛ367488900 │ 3
   ᴨ330952255947 Gnomon of cubic to be taken. 1150569                    19170 │ 3
    76060628419(1917639                      36773048169 Sq.       1102466700 │ 6
                                             §36771897600 │ 3          57510 │ 36
                                                  17760 │ 3        6614800200 │ 216
                                             110315692800 │ 3         345060 │
                                                 575280 │ 9          172530. │
                                             330947078400 │ 27          216 │
                                                5177520 │          †6616870776 │
                                                     27 │
                                             ᴨ330952255947
```

[Table 4-12]

The corrected divisor in this example is seen, by preceding with the working, to be always increased from smaller to larger values : 783, 79443, 8024667, 803156928, 80319.

It is by this divisor, the figure to be found placed in the quotient, which will give the gnomon of the cubic, etc. But with that divisor becoming small, the other places towards the left are unaffected, and we can use this for that divisor, in the usual way, by neglecting the gnomon, and the quotient will be for the same subsequent total figures, clearly the same which the other way of working would arrive at[9]. For:

```
7606028419
  3  .  ..  ..  ..      To be added to the given divisor
110319               The square of the triple to be taken away
80319  .  .  .        Corrected divisor.   *
```

```
76060628419 (94698          *This divisor by the rule of common
722871                       division gives the quotient 94698,
  377352                     as we see here. Which ought to be
  321276                     9469736386. The total therefore of
   560768                    the Subtended Chord sought is
   481914                    1917639469736386. Of which all
    788644                   the figures are found accurately, if
    722871                   we proceed with this method which
     656731                 from the beginning of the working
     652552                 we assume to be useful.
       141799
```

[Table 4-13]

Therefore, by the third conclusion of the Third Chapter, for any given subtended chord of an arc, we can find the subtended chord of the triple of the arc, and (if the arc triplicated should be more than the whole circumference) the subtended chord of the excess over the Circle. We can also find the subtended chord of a third of the given arc, and the subtended chord of the sum of the total of the circle and the arc given.

Notes on Chapter Four.

[1]   The phrase '*ab eodem fonte ...dimanet*' Lit. 'may spread (abroad) or (in different directions) from the same source or spring', of which Briggs was obviously fond, as he uses it in the *Arithmetica.*

[2]   This is the first time Briggs has used the ' = ' sign; He has set up cubic equations before in the Arithmetica Logarithmica, Chapter 29, when dealing with the diagonals of the regular heptagon and nonagon. The number in the circle refers to the power of the variable, not given a letter by Briggs, but understood implicitly to be present. Thus, Briggs' cubics are : $3x = 0.61803398875 + x^3$, or $3x - x^3 = 0.61803398875$, corresponding to Note 2 of Chapter 3, or $3x = A + x^3$ in general, where $0 < A < 2$. It is convenient to write $f(x) = x^3 -3x +A$. As x is 'small' in this case, it is legitimate to neglect the cubed term initially, and take the first approximation x_1 to be $A/3$; or $x_0 = 0$. Thus begins one of the great tasks performed by Briggs in this book: the setting up of the iterative method we have come to know as Newton's Method, for the solution of polynomial equations. This eventually leads to an immaculate table of sines at $^5/_8$ ^{0} spacing to 22 decimal places [Table 13-2], his answer to Ptolemy's Table of Sines. This table would seem to be the basis for all other tables of sines thereafter.

[3]   The Latin or the English *gnomon* is the pin of the sundial, which casts the shadow. Thus, the calculation 'points' to the next number to insert in the iteration: for Briggs uses the term as the name of the little procedure he performs in a trial manner to produce the correction to the numerator $f(x_n)$ following an iteration: we will retain this name, rather as you might have a special name for a subroutine in a computer program, to which Briggs' gnomon is similar.

[4]   Briggs solves the above cubic f(x) by an iterative approach which is now called Newton's method, or the Newton-Raphson method, for finding an approximate root of f(x). The origins of the method seem to be obscure, but it was known to the early Arab mathematicians: Viete certainly used a method for extracting cube roots in an iterative manner, that may be found in a translation of some of Viete's *Opus Restitutae Mathematicae Analyseos, seu Algebra Nova* by T. Richard Witmer, *The Analytic Art,* (The Kent State U.P., 1983, p.311 onwards): this work suffers from a lack of commentary! Viete's method *is* commented on by Goldstine, *A History of Numerical Analysis...*(Springer-Verlag, (1977), p.66), while Whiteside informs us that Newton was unaware of the existence of the *Trigonometria Britannica*! For whatever reasons, the present exposition by Briggs has been almost completely ignored by commentators on the history of Newton's method. The interested reader may consult this article for some further information : '*Henry Briggs: The Trigonometria Britannica'* by Ian Bruce, in the *Math. Gazette*, Vol. 88, No. 513, Nov. 2004 As it appears, Briggs amended the method of Viete so that it becomes identical with the modern method. In addition, the business of cutting an arc into a number of equal smaller arcs and relating their chord lengths had been attended to by Viete and Anderson (a grand uncle of James Gregory of telescope fame, then Professor of Mathematics in Paris), *Ad Angulares Sectiones,* to be found in the *Opera Mathematica* of Francois Viete, p.287 - 304; (Olms, 1970). Again, Briggs has done this in his own way, but arrives at equations the same as those of Viete and Anderson, which he does not acknowledge or may not have realised.

    According to this method, if x_1 is an approximation for the root then a better approximation x_2 is given by: $x_2 = x_1 - f(x_1)/f'(x_1)$; there are situations where this

algorithm does not converge. However, in the present situation, there is no difficulty. It is appropriate to proceed through Briggs' calculations bearing this correction mechanism in mind:

1st iteration: $x_0 = 0$, and $x_1 = x_0 + \dfrac{x_0^3 - 3x_0 + A}{3 - 3x_0^2} = A/3 = 0.2$ (to 1 sig. fig.) ;

2nd iteration: $x_1 = 0.2$, and

$$x_2 = x_1 + \frac{x_1^3 - 3x_1 + A}{3 - 3x_1^2} =$$

$$0.2 + \frac{0.2^3 - 3.(0.2) + 0.618033988}{3 - 3.(0.2)^2} = 0.2 + \frac{0.008 - 0.6 + 0.618}{3 - 0.12} = 0.2 + \frac{0.026033}{2.88} = 0.209..$$

Note that Briggs gets to this stage in two moves; first he corrects the dividend, then he corrects the divisor. Now the calculations have become more time consuming, and Briggs has evolved a scheme for making the corrections to the remainder or dividend and the divisor. For argument's sake, consider the above iterations, and let $x_2 = x_1 + \Delta$, where $\Delta = 0.009$, found by inspection; then

$$x_3 = x_2 + \frac{x_2^3 - 3x_2 + A}{3 - 3x_2^2} = x_2 + \frac{(x_1 + \Delta)^3 - 3(x_1 + \Delta) + A}{3 - 3(x_1 + \Delta)^2} = x_2 + \frac{f(x_1) + g(x_1, \Delta) - 3\Delta}{f'(x_1) - 6x_1\Delta - 3\Delta^2}, \text{ where } f(x_1) \text{ has}$$

already been found, and the gnomon $g(x_1, \Delta)$ is defined by:

$g(x_1, \Delta) = 3x_1^2\Delta + 3x_1\Delta^2 + \Delta^3$, and 'points' to the next correction. The gnomon for Table 4-2 corresponds to $g(0.2, 0.009) = 3(0.2)^2(0.009) + 3(0.2)(0.009)^2 + (0.009)^3 = 0.00108 + 0.0000486 + 0.000000729 = 0.0011293$; hence, $f(0.2) + g(0.2, 0.009) - 3(0.009) = 0.026033988 + 0.0011293 - 0.027 = 0.000163317750$, as required for the dividend. In the case of the divisor, the modified divisor is $f'(0.2) - 6(0.2)(0.009) - 3(0.009)^2 = 2.88 - 0.0108 - 0.000243 = 2.8689$. Thus, there is no doubt about the method being used. These are the computations that occupy a lot of the space in the table, set out methodically with squares, cubes, etc, of the approximate root Δ, and with binomial coefficients that multiply these to give the correction gnomon on addition which is carried across to the point of application for the next division.

[5] There is a typographical error here, where addition is mentioned while it is actually subtraction, which has been corrected. Numerous numerical errors in the tables, where clearly wrong numbers have been inserted by the typesetter, have been corrected without comment throughout the translation.

[6] Note that cubics of the form $f(x) = x^3 - 3x + A$ cut the vertical axis at A, and have maximum turning point at $x = -1$ and a minimum turning point at $x = 1$. Hence, if $0 < A < 2$, then there are three real roots. We seek the smaller positive root in case 2 outlined by Briggs. Recalling that Newton's Method 'works' by finding a tangent line that cuts the axis closer to the root than the original estimate, then this is not always possible, especially near a turning point. In the present case, where $A = 1.9318516525578$, the roots can be shown to be -1.9923893961812; 0.8452365234576; and 1.1471528727235, and the positive roots the two positive roots are close together. Now, $x_1 = A/3 = 0.6$ is not a good starting point here according to Briggs, for:

$x_2 = 0.6 + 0.547/1.92 = 0.88$; $x_3 = 0.88 + (-.0266)/.6768 = 0.84$: presumably he was unhappy with the approximations oscillating by first increasing and then decreasing. However, in this case the oscillation does settle down; he must have had experiences of cases where the oscillations either grew larger and larger, or they settled into a loop. Hence, he is unwilling to chance it, and opts for a region where the convergence is well-behaved.

[7]   Newton's method fails when the either the first or a successive approximation passes through a turning point, as $f'(x) = 0$ leads to a correction at infinity. Briggs is aware of this, though he has chosen the first approximation as $x_1 = 1$, for he has little choice in this case. Having done so, he evaluates $f(x_1)$, where he has chosen $f(x) = 3x - A - x^3$; Thus,

$f(x_1) = 3x_1 - A - x_1^3 = 3 - (1.9318516525578 + 1) = 0.068148347422$: see [Table 4-8]. Note that the function has been inverted, to give a positive dividend, which is maintained throughout. Briggs cannot yet use the predictive property of the method, and resorts to setting $x_2 = x_1 + \Delta$, where $\Delta = 0.1$; for a negative value of Δ would converge on the previous root. The new value of the dividend is constructed, see [Table 4-9]:

$f(x_1 + \Delta) = 3(x_1 + \Delta) - A - (x_1 + \Delta)^3 = f(x_1) + 3\Delta - 3x_1^2 - 3x_1 - \Delta^3$; and the divisor can now be amended:

068(11	$f(x_1)$ $(x_1 + \Delta)$
3	$3\Delta = 0.3$
368	$f(x_1) + \Delta$
§331. ..	$(3x_1^2 + 3x_1 + \Delta^3)$
37148	$f(x_2)$
36300	$3x_2^2$
3 . . .	
6300	$f'(x_2) = 3 - 3x_2^2$

Subsequently, a scheme similar to the former is implemented on $f(x)$ as defined here:

$$x_3 = x_2 + \frac{3x_2 - A - x_2^3}{3 - 3x_2^2} = x_2 + \frac{3(x_1 + \Delta) - A - (x_1 + \Delta)^3}{3 - 3(x_1 + \Delta)^2}$$

$$= x_2 + \frac{f(x_1) + 3\Delta - g(x_1, \Delta)}{f'(x_1) - 6x_1\Delta - 3\Delta^2}$$

The working in the tables has been put into an abbreviated form, with the final root shown extracted.

[8]   Given $(2R)sin(3\theta/2) + (2R)^3 sin^3(\theta/2) = 3(2R)sin(\theta/2)$, as from note 2, Chapter 3. Then with $2R = 1$, then $3\theta = 81^0 + 360^0 = 441^0$, and $\theta = 147^0$; $2sin(3\theta/2) + (2sin(\theta/2))^3 = 3(2sin(\theta/2))$, or $2sin(3\times147/2) + p^3 = 3p$, where $p = 2sin(147/2)$ is to be found. However, $2sin(441/2) < 0$, and the magnitude is given to be 1.29889: hence the equation shown.

[9]   The method eventually gives several more correct places than the one sought at the left: in this way, the scheme is self-correcting, a great bonus to anyone involved in numerical analysis by hand. We show more of this with the quintic equations, where spreadsheet analysis brings out this feature.